DOCUMENT RESUME

ED 090 013 SE 017 566

AUTHOR	Thompson, Russ; Fuller, Albert
TITLE	Algebra I, Package 03-02, The Language of Algebra.
INSTITUTION	Arnold Public Schools, Nebr.
SPONS AGENCY	Bureau of Elementary and Secondary Education (DHEW/OE), Washington, D.C.
PUB DATE	72
NOTE	32p.; For related documents, see SE 017 553 through 565 and SE 017 567 through 575
EDRS PRICE	MF-$0.75 HC-$1.85 PLUS POSTAGE
DESCRIPTORS	*Algebra; Grade 9; Individualized Instruction; *Instructional Materials; Mathematical Concepts; Mathematical Vocabulary; Objectives; *Secondary School Mathematics; Symbols (Mathematics); *Teaching Guides; *Tests
IDENTIFIERS	Elementary Secondary Education Act Title III; ESEA Title III

ABSTRACT

This teacher guide is part of the materials prepared for an individualized program for ninth-grade algebra and basic mathematics students. Materials written for the program are to be used with audiovisual lessons recorded on tape cassettes. For an evaluation of the program, see ED 086 545. In this guide, the teacher is provided with objectives for each topic area and guided to materials written for a given topic. Three short criterion tests are included for each topic covered. The content of this package centers on the language of algebra. The concept of a variable is develcped and solution sets are found for simple equations. Provided is practice in the use of exponents and quantifiers. This work was prepared under an ESEA Title III contract. (JP)

ED 090 013

U S DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

ALGEBRA I

Package # 03-02

THE LANGUAGE OF ALGEBRA

Prepared by

Russ Thompson and Albert Fuller

Under a Grant From
ESEA, Title III
Nebraska State Department of Education
Jack Baillie, Administrator
to
Arnold Public Schools, Arnold, Nebraska

© ARNOLD PUBLIC SCHOOLS 1972

PERMISSION TO REPRODUCE THIS COPYRIGHTED MATERIAL HAS BEEN GRANTED BY
Russell Thompson
Albert Fuller
TO E.RIC AND ORGANIZATIONS OPERATING UNDER AGREEMENTS WITH THE NATIONAL INSTITUTE OF EDUCATION. FURTHER REPRODUCTION OUTSIDE THE ERIC SYSTEM REQUIRES PERMISSION OF THE COPYRIGHT OWNER.

SE 017 566

The Language of Algebra

There are many special symbols and terms used in Algebra. You will be introduced to many of them in this package -- some you are already familiar with. You could learn to accomplish all of the operations of algebra without knowing some of the terms found in this package. You could also live a full life with a very limited vocabulary in the English language. However, a person can avoid many misunderstandings and can feel much more confident in himself if he knows the meanings of words used by a person he is conversing with.

Therefore, you are urged to take time to fully understand each of the symbols and terms introduced in this package. If you will do this, it will be much easier for you to talk to your friends and your teacher about algebra.

PACKAGE GOAL: to gain an adequate understanding of the language of algebra.

PACKAGE OBJECTIVES:

1. Given an expression that does not contain exponents and given the value of each variable that appears in it, state the number of terms in the expression and evaluate the expression.

2. Given an expression involving exponents and given the value of each variable that appears in it, evaluate the expression.

3. Given an expression in which some symbols of grouping have been omitted and given the value of each variable that appears in the expression, simplify the expression.

4. Given an open sentence and a replacement set, specify the solution set.

5. Given a statement containing a quantifier, indicate whether the statement is true or false and justify your answer.

6. Given a numerical relationship described in words and described by symbols and numbers, show how the symbols and numbers description is obtained from the word description.

I. U. # 03-02-01

Variables

You will need to recall:

Names, or symbols, for numbers are called numerical expressions.
Examples: $3 + 1$, $5 - 2 \div 3$, $4 \times 8 - 3$.

Changing the numeral by which a number is named in an expression does not change the value of the expression.

OBJECTIVES:

1. Given a mathematical expression, state the number of terms of the expression.

2. Given a variable expression and the value of each variable, evaluate the expression.

3. Given an expression that does not contain exponents and given the value of each variable that appears in it, state the number of terms in the expression and evaluate the expression.

ACTIVITIES:

Study: S & M: pages 31-33 (objectives 1,2, & 3)

Suggested exercises:
page 34, written exercises, part A, odd.
pages 34-35, problems, part A, odd.

Criterion Test 03-02-01-01

1. State the number of terms in each expression.

   (a) $x - bc$ (c) $(x - 2c)(x + 2c)$
   (b) $(xy)(ab)$ (d) $(x - 2c)\ x + 1$

2. Evaluate each of the following expressions.

   (a) $\frac{bx + c}{2x + a}$, if a is 5, b is 0, c is 4, and x is 3.5

   (b) $1/2h\ (b + c)$: Area of a trapezoid

   let h be 16 ft., b be 40 ft. and c be 15 ft.

3. State the number of terms in each expression and evaluate it. a is 2, b is 1, c is 3, x is 0, y is 4, and z is 1/2.

   (a) abcxyz (b) $(x + b)(y - 2a)$ (c) $a + b + c$

Criterion Test 03-02-01-02

1. State the number of terms in each expression.

   (a) $b - xy$ (c) $(x - y)(x + 1)$
   (b) $(ab)(cd)$ (d) $x + 2(x + 1)$

2. Evaluate each of the following expressions.

   (a) $\frac{x + 2c}{2b - a}$, if a is 8, b is 5, c is 7, and x is 4.

   (b) area of a triangle: $1/2bh$.
   let b = 8.5 inches and h = 4 inches

3. State the number of terms in each expression and evaluate it. a is 2, b is 1, c is 3, x is 0, y is 4, and z is 1/2.

   (a) abxyz (b) $(c - b)(yz)$ (c) $ac - yz$

Criterion Test 03-02-01-03

1. State the number of terms in each expression.

   (a) $a + b\ (c + d)$
   (b) $(x + y)(3 + x)$
   (c) $(x - 4)(ab)$
   (d) $ab - xy + 3$

2. Evaluate each of the following expressions.

   (a) $\frac{2x + a}{2b - c}$, if a is 5, b is 4, c is 3, and x is 0.

   (b) Perimeter of a rectangle: $2\ell + 2w$
   let ℓ be 4 yards and w be 1.5 yards

3. State the number of terms in each expression and evaluate it. a is 2, b is 1, c is 3, x is 0, y is 4, and z is 1/2.

   (a) acxyz
   (b) $(y - x)(a - b)$
   (c) $cy - az$

Answers to Criterion Tests

Test 03-02-01-01

1. (a) 2 (b) 1 (c) 1 (d) 2

2. (a) 1/3 (b) 440 sq. ft.

3. (a) 1, 0 (b) 1, 0 (c) 3,6

Test 03-02-01-02

1. (a) 2 (b) 1 (c) 1 (d) 2

2. (a) 9 (b) 17 sq. in.

3. (a) 1, 0 (b) 1, 4 (c) 2, 4

Test 03-02-01-03

1. (a) 2 (b) 1 (c) 1 (d) 3

2. (a) 1 (b) 11 yards

3. (a) 1, 0 (b) 1, 4 (c) 2, 11

I. U. # 03-02-02

Exponents

You will need to recall:

A positive integer is a member of the set {1,2,3,4, ...}.

OBJECTIVES:

1. Given a term, state the coefficient, the base, and the exponent.

2. Given an expression in expanded form, write it in exponential form.

3. Given an expression involving exponents and given the value of each variable that appears in it, evaluate the expression.

ACTIVITIES:

Study: S & M: pages 36-38 (objectives 1-3)

If asked to define an exponent, you must be very careful what you say. An exponent does not "tell how many times a number is multiplied by itself."

Consider 5^2. If I take 5 times itself twice ($5 \times 5 = 25$ and $5 \times 5 = 25$), I get 25 and 25. This is incorrect. $5^2 = 25$.

Consider 4^3. If I take 4 times itself three times ($4 \times 4 = 16$, $4 \times 4 = 16$, $4 \times 4 = 16$), I get 16,16, and 16. This is incorrect. $4^3 = 4 \times 4 \times 4 = 64$.

Suggested exercises:

S & M: page 38; ex. 13-18 (objective 1)
page 39; ex. 1-15 odd, (objective 2)
page 39; ex. 17-27 odd, (objective 3)
page 39; problems 1,3,5 (objective 3)

Criterion Test 03-02-02-01

1. For each term, state the coefficient, the base, and the exponent in that order.

   (a) $2xy^2$
   (b) $6y^4$
   (c) $3(x - y)^2$

2. Write in exponential form.

   (a) $c \cdot c \cdot c \cdot c$
   (b) the fourth power of $x + 3$.
   (c) $12 \cdot x \cdot y \cdot y \cdot y$

3. Evaluate each expression for the given values of the variables.

   (a) $2x^3$; 2
   (b) $2(xy)^2$; x is 5, y is 2.
   (c) $(3x)^2$; 4
   (d) Surface area of a prism: $2w^2 + 4\ell w$
   let ℓ be 4 ft. and w be 3 ft.

Criterion Test 03-02-02-02

1. For each term, state the coefficient, the base, and the exponent, in that order.

   (a) $3xy^3$
   (b) $7y^2$
   (c) $4(a + b)^3$

2. Write in exponential form.

   (a) $x \cdot x \cdot x \cdot x \cdot x \cdot x$
   (b) the cube of $x - 2$
   (c) $3 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$

3. Evaluate each expression for the given values of the variables.

   (a) $3x^2$; 4
   (b) $2(xy)^3$; x is 2, y is 3.
   (c) $(2x^3)$; 3
   (d) Volume of a sphere: $4/3\ \pi r^3$
   let r be 6 inches. $\pi \triangleq 22/7$

Criterion Test 03-02-02-03

1. For each term, state the coefficient, the base, and the exponent, in that order.

   (a) $4xy^3$
   (b) $5y^3$
   (c) $2(x - y)^3$

2. Write in exponential form:

   (a) $x \cdot x \cdot x$
   (b) the square of $x + 3$
   (c) $3 \cdot x \cdot y \cdot y \cdot y \cdot z$

3. Evaluate each expression for the given values of the variables.

   (a) $3x^2$; 4
   (b) $2(xy)^3$; $x = 2$, $y = 3$
   (c) $(4x)^2$; 3
   (d) Volume of a right circular cylinder: $\pi r^2 h$
   let r be 4 inches, h be 35 inches, and $\pi \doteq 22/7$

Answers to Criterion Tests

Test 03-02-02-01

1. (a) 2, y, 2
   (b) 6, y, 4
   (c) 3, x - y, 2

2. (a) c^4
   (b) $(x + 3)^4$
   (c) $12 \cdot x \cdot y^3$

3. (a) 16
   (b) 200
   (c) 144
   (d) 66 sq. ft.

Test 03-02-02-02

1. (a) 3, y, 3
   (b) 7, y, 2
   (c) 4, a + b, 3

2. (a) x^6
   (b) $(x - 2)^3$
   (c) $3x^2y^4$

3. (a) 48
   (b) 432
   (c) 54
   (d) 905 1/7 cubic inches

Test 03-02-02-03

1. (a) 4, y, 3
   (b) 5, y, 3
   (c) 2, x - y, 3

2. (a) x^3
   (b) $(x + 8)^2$
   (c) $3xy^3z$

3. (a) 48
   (b) 432
   (c) 144
   (d) 1760 cubic inches

I. U. # 03-02-03

Order of Operations

OBJECTIVES:

1. Given a numerical expression in which some symbols of grouping have been omitted, simplify the expression.

2. Given an expression in which some symbols of grouping have been omitted and given the value of each variable that appears in the expression, simplify the expression.

ACTIVITIES:

Study:
S & M: pages 41-42 (objective 1 and 2).

Suggested exercises:
S & M: page 43; 1-19 odd (objective 1).
page 43; 25-37 odd (objective 2).

Criterion Test 03-02-03-01

1. Simplify each expression.

   (a) $17 + 5\ (2 + 8) - 38$
   (b) $8 \cdot 2 - 2^2 + 3^2$
   (c) $4(9 + 1) - 35 \div 7$

2. Evaluate each expression if x is 2, y is 3, and z is 4.

   (a) $2y^2 - y + 1$
   (b) $\dfrac{x^2 + y - z}{y^2}$
   (c) $z^3 - xy + 5$

Criterion Test 03-02-03-02

1. Simplify each expression.

   (a) $14 + 3\ (2 + 7) - 4$
   (b) $7 \cdot 3 - 3^2 \div 4 \div 2$
   (c) $3(6 + 3) - 21 \div 3$

2. Evaluate each expression if x is 2, y is 3, and z is 4.

   (a) $3x^2 - x - 2$
   (b) $\dfrac{2x^3 - 2xy + z}{xyz}$
   (c) $5x^2y + z$

Criterion Test 03-02-03-03

1. Simplify each expression.

   (a) $14 + 3(8 + 5) - 12$
   (b) $6 \cdot 3 - 3^2 + 4^2$
   (c) $48 \div 8 + 3(8 + 2)$

2. Evaluate each expression is x is 2, y is 3, and x is 4.

   (a) $3x^2 + 3y^2 + z^2$
   (b) $\dfrac{x^2 + xy + 5}{3xz}$
   (c) $x^3 + xy + 2$

Answers to Criterion Tests

Test 03-02-03-01

1. (a) 29
   (b) 21
   (c) 35

2. (a) 16
   (b) 1/3
   (c) 63

Test 03-02-03-02

1. (a) 37
   (b) 14
   (c) 20

2. (a) 8
   (b) 1/3
   (c) 64

Test 03-02-03-03

1. (a) 41
   (b) 25
   (c) 36

2. (a) 55
   (b) 5/8
   (c) 16

I. U. # 03-01-04

Open Sentences

You need to recall:

The domain (replacement set) of a variable is the set of numbers that the variable may represent.

OBJECTIVES:

1. Given an open sentence and a replacement set, substitute members of the replacement set in the open sentence and tell which of the resulting statements are true.

2. Given an open sentence and a replacement set, specify the solution set of each equation by roster and each inequality by graph.

ACTIVITIES:

Study:
S & M: pages 44-45 (objectives 1 and 2)

Suggested exercises:
S & M: page 46; ex. 1-11, odd (objective 1)
pages 46-47; ex. 13-35, odd (objective 2)

Criterion Test 03-02-04-01

1. Substitute each member of the given set in the open sentence and tell which of the resulting statements are true.

   (a) $x > 3$; {3,4,5}
   (b) $x \neq 5$; {3,4,5,6}
   (c) $y^2 > 2$; {0,1,2}

2. Specify the solution set of each equation by roster and each inequality by graph.

   (a) $x - 2 = 10$ {the whole numbers}
   (b) $x + 2 < 7$ {the positive integers}
   (c) $21 < 7x$ {the positive numbers}
   (d) $14 = 5x - 1$ {the real numbers}

Criterion Test 03-02-04-02

1. Substitute each member of the given set in the open sentence and tell which of the resulting statements are true.

   (a) $y > 4$; {3,4,5}
   (b) $y \neq 7$; {6,7,8}
   (c) $x^2 > 3$; {1,2,3}

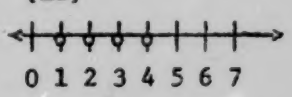
2. Specify the solution set of each equation by roster and each inequality by graph.

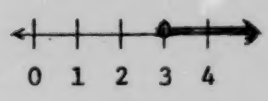
   (a) $x - 3 = 10$ {the whole numbers}
   (b) $3x > 21$ {the positive integers}
   (c) $x + 5 < 7$ {the positive integers}
   (d) $2x = 10$ {the real numbers}

Answers to Criterion Tests

Test 03-02-04-01

1. (a) true for 4 and 5
   (b) true for 3,4, and 6
   (c) true for 2

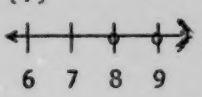
2. (a) {12}
   (b) 

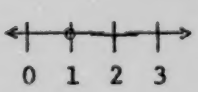
   (c) 

   (d) {3}

Test 03-02-04-02

1. (a) true for 5
   (b) true for 6 and 8
   (c) true for 2 and 3

2. (a) {7}
   (b) 

   (c) 

   (d) {5}

Criterion Test 03-02-04-03

1. Substitute each member of the given set in the open sentence and tell which of the resulting statements are true.

   (a) $x < 4$; {3,4,5}
   (b) $x \neq 3$; {3,4,5}
   (c) $x^2 < 20$ {3,4,5}

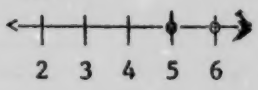
2. Specify the solution set of each equation by roster and each inequality be graph.

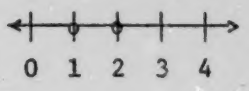
   (a) $x + 2 = 8$ {the whole numbers}
   (b) $20 < 5x$ {the positive integers}
   (c) $7 > x + 4$ {the positive integers}
   (d) $5x + 2 = 17$ {the real numbers}

Answers to Criterion Tests (continued)

Test 03-02-04-03

1. (a) true for 3
   (b) true for 4 and 5
   (c) true for 3 and 4

2. (a) {6}
   (b) 

   (c) 

   (d) {3}

I. U. # 03-02-05

Quantifiers

OBJECTIVES:

1. Given a statement containing a quantifier, indicate whether the statement is true or false and justify your answer.

ACTIVITIES:

Study:
S & M: pages 48 - 49 (Objective 1)

Suggested exercises:
S & M: page 49; exercises 1 - 16 (Objective 1)
page 50; exercises 1 - 8 (Objective 1)

Criterion Test 03-02-05-01

1. Indicate whether the statement is true or false and justify your answer.

   (a) All numbers are negative.

   (b) The square of each real number k is greater than k.

   (c) Any natural number n is less than 2n.

   (d) There is a number t such that $2t + 1 = 8$.

Criterion Test 03-02-05-02

1. Indicate whether the statement is true or false and justify your answer.

   (a) All cats are black.

   (b) Some numbers are not whole numbers.

   (c) There exists an integer y such that $2y + 1 > 6$.

   (d) All numbers are greater than 0.

Criterion Test 03-02-05-03

1. Indicate whether the statement is true or false and justify your answer.

   (a) Every polygon has four sides.

   (b) Some triangles have three sides congruent.

   (c) For every natural number s, $s + 2 > 2$.

   (d) There exists a number x such that $x^2 < x$.

Answers:

Criterion Test 03-02-05-01

1. (a) false; 5 is a number and 5 is not negative.

   (b) false; $\frac{1}{2}$ is a real number and $(\frac{1}{2})^2$ is not greater than $\frac{1}{2}$.

   (c) false; 0 is a natural number and 0 is not less than $2 \cdot 0$.

   (d) true; 3.5 is a number and $2(3.5) + 1 = 8$.

Criterion Test 03-02-05-02

1. (a) false; there is a white cat.

   (b) true; $\frac{1}{2}$ is a number but not a whole number.

   (c) true; 4 is an integer and $2 \cdot 4 + 1 > 6$.

   (d) false; -5 is a number that is not greater than 0.

Criterion Test 03-02-05-03

1. (a) false; a triangle is a polygon that does not have four sides.

   (b) true; an equilateral triangle has 3 sides congruent.

   (c) false; 0 is a natural number and $0 + 2$ is not greater than 2.

   (d) true; $\frac{1}{2}$ is a number and $(\frac{1}{2})^2$ is less than $\frac{1}{2}$.

I. U. # 03-02-06

Applying Mathematical Expressions and Sentences

OBJECTIVES:

1. Given a word description of a number, write it in terms of a variable.

2. Given a numerical relationship described in words and described by symbols and numbers, show how the symbols and numbers description is obtained from the word description.

ACTIVITIES:

Study:
S & M: pages 50 - 51 (Objectives 1 and 2)

Suggested exercises:
S & M: pages 51 - 52; oral exercises 1 - 19, odd. (Objective 1)

page 53; exercises 1 - 10 (Objective 2)

Please use the red answer book when checking your answers. The answers in the back of your book are not complete enough for these exercises. This applies to exercises on page 53 only.

Criterion Test 03-02-06-01

1. Express the number described in terms of n.

   (a) a number that is two less than n.
   (b) If Bob was n years old 5 years ago, how old is he now?
   (c) How much will n 8-cent stamps cost?
   (d) If n is an even number, what is the next larger even number?

2. Show how the given open sentence can be obtained from the word description.

   (a) $k + 5 = 23$
   Five years from now Jim will be 23 years old.

   (b) $x + (x + 2) + (x + 4) = 24$
   The sum of three consecutive even integers is 24.

Criterion Test 03-02-06-02

1. Express the number described in terms of n.

   (a) a number is five more than n.
   (b) If Laurie is n years old now, how old was she 5 years ago?
   (c) If n cigars cost $1.50, what does one cigar cost?
   (d) If n is an integer, what is the next smaller integer?

2. Show how the given open sentence can be obtained from the word description.

   (a) $2x + 1 = 15$
   Barbara is 15. She is one year more than twice as old as Jimmy.

   (b) $20 - x = 15$
   In a class of 20 students there are 15 girls.

Criterion Test 03-02-06-03

1. Express the number described in terms of n.

   (a) a number twice as large as n.
   (b) If Ruth is n years old now, how old will she be six years from now?
   (c) If a bottle of pop costs 15¢, what will n bottles cost?
   (d) What is the number that is three less than the square of n?

2. Show how the given open sentence can be obtained from the word description.

   (a) $2\ell + 2(\ell - 5) = 120$
   A rectangle whose width is 5 less than its length has a perimeter of 120.

   (b) $x + 4 = 2(x - 16)$
   Four years from now Rick will be twice as old as he was sixteen years ago.

Answers:

Criterion Test 03-02-06-01

1. (a) $n - 2$
   (b) $n + 5$
   (c) $8n¢$
   (d) $n + 2$

2. (a) let k = Jim's age now. In five years he will be $k + 5$. He will be 23, so $k + 5 = 23$.
   (b) let x = the smaller integer. The other two would be $x + 2$ and $x + 4$. The sum would be $x + (x + 2) + (x + 4)$. Since this sum is 24, $x + (x + 2) + (x + 4) = 24$.

Criterion Test 03-02-06-02

1. (a) $n + 5$
   (b) $n - 8$
   (c) $\frac{150}{n}$ ¢
   (d) $n - 1$

2. (a) let x = Jimmy's age. Then $2x$ would be twice his age. $2x + 1$ would be one more than twice his age. Since $2x + 1$ and 15 are both expressions for Barbara's age, $2x + 1 = 15$.
   (b) let x = the number of boys. Then $20 - x$ would be the number of girls. Since there are 15 girls, $20 - x = 15$.

Criterion Test 03-02-06-03

1. (a) $2n$
   (b) $n + 6$
   (c) $15n$ ¢
   (d) $n^2 - 3$

2. (a) let ℓ = the length. Then $\ell - 5$ would be the width. 2ℓ would be two lengths. $2(\ell - 5)$ would be two widths. $2\ell + 2(\ell - 5)$ would be the perimeter. Since the perimeter is also 120, $2\ell + 2(\ell - 5) = 120$.
   (b) let x be Rick's age now. In four years he will be $x + 4$. Sixteen years ago he was $x - 16$. Twice his age then was $2(x - 16)$. Therefore, $x + 4 = 2(x - 16)$.

END OF PACKAGE # 03-02